REFLEXIVE DECISION MODEL

1. Basic Decision Graph and Model

1.1. **Two guiding principles.**

(1) No probability for acts.
(2) Acts are exogenous.

Spohn argues that the second principle which is widely seen in the decision theoretic literature is compatible with his first principle. This is reflected in the Bayesian nets that the action nodes are uncaused or parentless.

1.2. **Bayesian net with action nodes.** The model is based on standard causal graph $< U, \rightarrow >$ and Bayesian net with added action nodes.

Decision graph: A *basic decision graph* $< U, \rightarrow, H >$ is a causal graph $< U, \rightarrow >$ with a set H of exogenous action nodes of U.

Decision model: A *basic decision model* is a structure $< U, \rightarrow, H, (p_h), u >$ where $< U, \rightarrow, H >$ is a decision graph, (p_h) is a family of probability measure for the given set W of occurrence variables $(W = U - H)$ agreeing with $< U, \rightarrow >$, and u is utility function from $\bigtimes U$ into $\mathbb{R}$.

Decision rule: Choose a course of action $h \in \bigtimes H$ that maximizes expected utility:

$$\sum_{w \in \bigtimes W} u(h, w) \cdot p_h(w).$$

- An important consequence is that in a basic decision model all non-descendants of an action variable are probabilistically independent of it. This is entailed by the exogeneity of action variables, as is easily verified with the help of d-separation.
- The common aim of CDTs is to find a representation in which states or variables causally independent from the actions are also probabilistically independent and which use such probabilities for calculating the agent's expected utilities. p.111

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2. Reflexive Decision Graph and Model

2.1. **Strategies and reflexion.** Two ways of doubting the two principles above

(1) On the one hand, the agent himself may make his actions dependent on the behavior of other variables and thus turn the action variables into endogenous ones; this is what is called **strategic behavior**. By deciding for a certain strategy the agent obviously accepts certain probabilities for the actions covered by the strategy, in contradiction to the two principles.
   - This way of *internalizing* acts as endogenous parameters by adding decision/intention nodes to Bayesian nets is what is behind the reflexive decision graph and model described below.
   - Instead of individual acts, what is being chosen here are so-called dependency schemes, which are possible plans for acting under different contingencies.
   - In a causal graph, these are the decision/intention nodes. "Each action node is preceded by a decision node, and then to define a feasible strategy as a dependency scheme which makes each action node depend only on its associated decision node. Obviously a decision node causally depends, in turn, on many other variables; thereby, the action nodes direct intentional dependence on the decision node ramifies into various indirect dependencies."
   - Then in this framework, actions (nodes) are considered as *mere implementations* of the dependency schemes chosen.

(2) On the other hand, it is hard to see why the agent should not be able to reflect on the causes of his own actions, just as he does concerning the actions of others. ... Why should the agent be unable to take doxastic attitudes like predicting, explaining, etc. toward his own actions, if he can very well do so toward the actions of others? One should indeed think that he is particularly well endowed in his own case because he has so much more data about himself than about anybody else. p.114
   - The agent is in a better position to know his/her habits, convention, anxiety and so on. These pieces of behavior may or may not be under the agent's rational control

- If they are, then the prediction is incomplete "unless it mentions that the particular instantiation of a habit or convention is confirmed by rational control." The reason for the particular instantiation of a habit or convention however cannot refer to the very habit or convention.
- If they are not, then they are not qualified for being actions.

2.2. **Bayesian net with decision nodes.** The model is based on the basic decision graph and basic decision model introduced above with added decision nodes.

Reflexive decision graph: A *reflexive decision graph* $< U, \rightarrow, H, D >$ is a causal graph $< U, \rightarrow >$ with/satisfying

(1) a set H of exogenous action nodes of U,
(2) a set D of decision nodes,
(3) a set $W = U - (H \cup D)$ of occurrence variables,
(4) each action node has exactly one decision node as the only parent, and each decision node has at least one action node as child. p.115

- Spohn rejects the following questions. p.115
  (1) One might require that no two action nodes have the same parental decision node.
  (2) One might require that each action node be immediately preceded in time by its parental decision node.
  (3) The explication allows a decision node to have other children than action nodes, but one might wonder how this is possible.

Reflexive decision model: A *reflexive decision model* $\hat{\delta}$ is a structure $< U, \rightarrow, H, D, p, u >$ where $< U, \rightarrow, H, D >$ is a reflexive decision graph, p is a probability measure for U, and u is utility function from $\bigtimes U$ into $\mathbb{R}$.

- The fact that p unrestrictedly distributes over the whole of U, in contrast to the probabilities in basic decision models, reflects the point that in the reflexive perspective there is no such restriction; the agent now has beliefs about his own actions and even about his own decision situations

2.3. **Shadow arrow.** Further constraints are given to supplement the reflexive decision model.

Self-knowledge (SK): If $\Delta_0 \in D$ is temporarily the first decision node, there is a particular $\delta_0 \in \Delta_0$ such that $p(\delta_0) = 1$.

- Δ_0 indexes the timing of a given reflexive decision graph.
- At that very time the agent knows in which decision situation he presently finds himself. He may not have foreseen it, and he may have forgotten it later on; but at the time of decision he knows his subjective view of his situation; and the model represents. only this view

Truncated reduction (TR): The basic decision model δ_0 is the truncated reduction of $\hat{\delta}$ by the first decision node Δ_0.

- The causal graph $< U - \{\Delta_0\}, \rightarrow^{tr} >$ is obtained from $< U, \rightarrow >$ by deleting, together with Δ_0, all arrows ending or starting at Δ_0 and, provided $Oc(\Delta_0)$ (other children) is not empty, by adding arrows from all $A \in Ac(\Delta_0)$ (action child) and all $B \in Pa(\Delta_0)$ to all $C \in Oc(\Delta_0)$.
- The action nodes in $Ac(\Delta_0)$ are thereby turned into exogenous variables, and the other children of Δ_0, if any, become directly causally dependent on all the parents *and* all the action children of Δ_0.

References

Spohn, W. (2012). Reversing 30 years of discussion: Why causal decision theorists should one-box. *Synthese 187*(1), 95–122.