

## ACYCLICITY PRINCIPLE AND LOGIC OF CHOICE

### 1. CASSANDRA'S PARADOX

**Example.** Eve is asked by Clive to lend him, on short notice and without security, \$1000, which he urgently needs to complete a deal. He tells her that he will receive in a couple of weeks a large sum and will return the loan with additional \$500 interest. Eve postpones her answer and deliberates. Given her beliefs at that stage, she inclines to grant his request. Later, at a party, she meets Cassandra, known for psychological insights and successful predictions of people's behavior. Cassandra makes two predictions:

$P_1$ : Clive will not keep the promise.

$P_2$ : Eve will lend the money to Clive.

Eve has two available options:

$L_1$ : Lend the money to Clive.

$L_2$ : Do not lend the money.

The question is what Eve shall do after learning Cassandra's second prediction ( $P_2$ ) which makes explicit reference to her choice?  $\triangleleft$

- Eve is facing a dilemma as she realizes whatever she chooses to do will seem to be the wrong choice.
- The choices are being considered in an expert system where the reliability of the expertise depends on the agent's choice, while, at the same time, the choices are made based on the expert's recommendations, hence the reasoning is *circular*.
- "Cassandra's second prediction has no place in Eve's deliberations. . . . Eve should reflect that Cassandra's second prediction did not state anything that could enter as a factor into the deliberation." p.125
- Some decision-theoretic principle is needed in order to block the part of reasoning that gives rise to the vicious circular.

### 2. THE PRINCIPLE

Let  $A$  be a description of a state of affair that the agent can bring about and  $R$  be the agent's reason for the choice. The *logic of choice* can be given by the scheme.

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**C-sentence:** I make  $\lceil A \rceil$  true, for reason  $R$ .

In light of the loop-forming-arguments in the examples given, the question is what should or should not enter the agent's deliberation in making the decision.

**ac:**  $R$  may refer to  $A$ , but it should not refer, directly or indirectly, to the choosing of  $A$ .

- The reasons  $R$  enter into the choosing process and it is they that determine whether some norm, such as rationality, is satisfied. ... In giving reasons for choosing  $A$ , we do as a rule refer to  $A$ . But we should not, on pain of creating a bad cycle, refer to the choosing itself. p.128
  - One can choose  $A$  for the sake of  $A$ , that is, for its intrinsic value. But  $A$  cannot be chosen for the sake of choosing  $A$ . p.129
  - If  $A$  is morally good, then to choose it for the right reason—*because* it is morally good—is to act morally. I can try to make  $A$  true, but I cannot try to make-for-the-right-reasons  $A$  true. Either my reasons are right or they are not. I can bring about  $A$ , but I cannot bring about my reasons.
- Special cases where my choosing of  $A$  enters my reasons for choosing  $A$ .
  - My choosing of  $A$  serves the purpose of *signaling* someone else of my choice.
  - I choose  $A$  in order to (a) avoid inactivity, (b) express a feeling of being in control, (c) impress someone else, (d) identify myself (as someone who makes this kind of choice).
  - But in all these case,  $A$  is *not* chosen for the sake of my choosing of  $A$ , but something else.

More generally,

**AC:** The reasons for choosing  $A$  can refer to each of the available options, but they cannot refer in an essential way to the *choosing* from these options, except through considerations of signaling.

### 3. PROBABILISTIC CASES

Since the reasons for making  $A$  true are instrumental in the choice, any non-trivial estimate of the probability, or the likeliness, of  $A$  being chosen by the agent, constitutes a reference to the choosing.

**AC\*:** One should not use conditional probabilities (or likeliness estimates) of choices, which are obtained by conditionalizing on some event upon which the choice, in the agent's judgment, has no bearing. p.131

- Terminology.
  - *Choice events:* events of the form  $A$  is made true (by the agent).

- *Given-world events*: events on which choices events have no bearing in the agent’s judgment.
- $AC^*$  amounts to a symmetric constraint, which prohibits any non-trivial probabilistic (or likeliness) connections between choice events and given-world events; such connections are banned because they enable nontrivial conditionalizations of the former on the latter.

As a condition on the prior probability,  $AC^*$  amounts to the following constraint

$AC^\#$ : Choice events and given-world events are probabilistically independent. Let  $X$  be a choice event and  $Y$  a given-world event,

$$X \text{ has no bearing on } Y \implies \Pr(Y|X) = \Pr(Y) \quad (3.1)$$

- The point of  $AC^\#$  is that the agent should decide the events upon which the choices have no bearing and then impose  $AC^\#$  as necessary condition on his or her prior probability.

#### 4. BAYESIAN CASSANDRA

The intuition (the C-sentence) behind the reliability of Cassandra’s prediction is that

- (In)**: The more successful Cassandra’s predictions have been, the more it is likely that her present prediction will succeed.
- Let  $B$  stand for her background information, which includes Cassandra’s both predictions  $P_1$  and  $P_2$ , where  $P_2$  is just  $L_1$ .
  - If  $B$  is considered, in the agent’s judgment, as a given-world event, then, in calculating probabilities, (3.1) mandates that

$$\Pr(L_1|B) = \Pr(L_1) \quad \Pr(L_2|B) = \Pr(L_2).$$

- $AC$  blocks  $P_2$  from entering (In), and hence prevents Eve from forming a circular reasoning

#### REFERENCES

- Gaifman, H. (1999). Self-reference and the acyclicity of rational choice. *Annals of Pure and Applied Logic* 96(1-3), 117 – 140.
- Jeffrey, R. C. (1983). *The Logic of Decision* (2nd ed.). Chicago: University of Chicago Press.