PROBABILITIES FOR ACTS

1. Jeffrey

1.1. **Bayesian betting interpretation.** Let X be a set of prizes and, to simplify matters, let x^* and x_* represent respectively the best and the worst prizes considered by you. Now you are invited to choose between the following two bets, where p is a proposition.

i. x^* if p; x_* if $\neg p$
ii. x^* if $\neg p$; x_* if p

The interpretation is that if you prefer (i) over (ii) then this reveals that you think it is more likely that p, and hence the *qualitative probability* with respect to p and q.

	p	$\neg p$
Bet i	x^*	x_*
Bet ii	x_*	x^*

Question: how to assign numerically precise probability?

Ramsey. The general method adopted by Ramsey (1926).

I. Probabity 1/2: Assume that p is an "ethically neutral" proposition then

$$\text{(i)} \sim \text{(ii)} \implies \mu(p) = 1/2$$

II. Utility: Using an ethically neutral p one can measure the utilities of prizes $x \in X$ through, again, the betting method: let the utility of x^* be 1 and x_* be 0, then for some $x \in X$,

iii. x
iv. x^* if p; x_* if $\neg p$.

$$\text{(iii)} \sim \text{(iv)} \implies u(x) = 1/2$$

Repeat the above process: the utility scale between x^* and x_* can be calibrated to arbitrary precision, then every prize in X can be assigned a numerical utility value.

$$\overset{1}{u(x^*)} \text{ —— } \cdots \text{ —— } \overset{1/2}{u(x)} \text{ —— } \cdots \text{ —— } \overset{1/4}{u(x')} \text{ —— } \cdots \text{ —— } \overset{0}{u(x_*)}$$

III. Subjective probabilities: For any proposition q,

v. x
vi. y if q; z if $\neg q$

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Given that the utility values of $x, y, z \in X$ are now known, one can calculate the probability of q if

$$\text{(v)} \sim \text{(vi)} \implies \mu(q) = \frac{u(x) - u(z)}{u(y) - u(z)}, \quad u(y) - u(z) > 0.$$

Fair betting rate. In general, under standard Bayesian betting interpretation, an agent is said to have degrees of belief r in a given proposition p, i.e., if $\mu(p) = r$, she should be indifferent between

(1) \$$r$
(2) \$1 if p; \$0 if $\neg p$.

1.2. **Jeffrey's generalization of Savage.** The inferential order in obtaining utilities and probabilities is reversed in Savage's theory:

P1-5		+ **P6**		+ **P7**
Qualitative probability	$\Rightarrow$	Quantitative probability Utility for simple acts	$\Rightarrow$	Utility for all acts

But Ramsey's betting framework is to a large extent preserved in Savage's system:

	good	rotten
break into bowl	6-egg omelet	all ruined
throw away	5-egg omelet one wasted	5-egg omelet

- In Savage (1972), the propositions that are assigned with probability assignments are (act-independent) state descriptions.
- In Jeffrey (1983), one can also assign probabilities to propositions that describe the agent's actions.

Example (The right wine)**.** The dinner guest who is to provide the wine has forgotten whether chicken or beef is to be served. He has no telephone, has a bottle of white and a bottle of red, and can only bring one of them since he is going by bicycle. The consequence matrix might well be the following.

	Chichen	Beef
Bring white	The right wine	The wrong wine
Bring red	An odd wine	The right wine

Which wine to bring? ◁

Jeffrey's solution.

	Chichen	Beef
White	1	-1
Red	0	1

	Chichen	Beef
White	$.75r$	$.25r$
Red	$.25(1-r)$	$.75(1-r)$

where .75 and .25 in the first row are conditional probabilities of chicken and beef given that white wine is served and that r **is the guest's subjective probability that he will bring it**. Jeffrey calculated: $U(R) = .75 > .5 = U(W)$.

2. Spohn

Thesis: *Any adequate quantitative decision model must not explicitly or implicitly contain any subjective probabilities for acts.*

2.1. **Probabilities for acts play no role in decision making.** "At no place does there enter any subjective probability for an act. The decision maker chooses the act he likes most..." (p.115)

- Spohn's remark echoes a point made by Savage.

Example (Car)**.** Jones is faced with the decision whether to buy a certain sedan for a thousand dollar, a certain convertible also for a thousand dollars, or to buy neither and continue carless. ◁

Savage's analysis.

Simple case: Choose the action according to the consequences they lead.

- "Chance and uncertainty are considered to have nothing to do with the situation" (Savage, 1972, p.83).

Complicated case: There are other contingencies under which the decision is being made.

- "Jones must take account of many uncertain future in actually making his choice. The relative fragility of the convertible will be compensated only if Jones's hope to arrange a long vacation in a warm and scenic part of the country actually materializes; Jones would not buy a car at all if he thought it likely that he would immediately be faced by a financial emergency arising out of the sickness of himself or of some member of his family; he would be glad to put the money into a car, or almost any durable goods, if he feared extensive inflation."
  - It is these other contingencies that are subject to the agent's probabilistic assessments, not the actions themselves.
- These led Savage to his *belief-act-consequence* model, where acts are taken as functions mapping from (act-independent) states to consequences, and it is the states over which the acts are defined that are the subject of uncertainty, not the acts themselves.

2.2. **The betting argument.** Suppose, in the wine example, the guest's subjective probability for his bring the white wine is r, i.e., $\mu(W) = r$, then, under the standard betting interpretation, he should be indifferent between the following two options:

(1) \$$r$
(2) \$1 if W; \$0 if $\neg W$.

In other words, he should be willing to pay a fee of r to accept the bet in exchange of a reward of \$1 on the event that he indeed is going to bring the white wine.

- However, for any $0 < r \leq 1$, the mere fact that the guest is willing to accept the bet of his bringing the white at the cost of r implies that he will be bringing the white wine *for sure*!
  - For, otherwise, it would be extremely unwise for him to *knowingly* pay a fee of r but actually bring a red wine to dinner while gaining nothing from the bet he paid for, where the loss can be easily avoided by simply rejecting the bet.
- Furthermore, if $r = 0$ then this means that the guest will bring the red wine *for sure*!
- The betting interpretation collapses when it comes to assigning subjective probabilities to acts.
- Use the frequentist interpretation?

Challenge: If the Bayesian betting interpretation fails for actions, then proponents of "probabilities for acts" need to provide a new interpretation for these probabilities, otherwise those numerical numbers assigned to acts are meaningless.

References

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